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Measuring Chaos

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Last summer I picked up a popular science book on chaos theory and really enjoyed it so I decided that this summer I would work for chaos theorist Deb King.

Throughout the vacation scholarship I was often asked about what chaos and entropy are so I will start with a brief explanation. Generally speaking, a system is chaotic if it has sensitive dependence on initial conditions. That is, I can change the starting conditions of the system by a tiny amount and get a widely different result. Entropy, meanwhile, is an ordering that mathematicians place on systems which, in a loose sense, attempts to measure how 'complex' or chaotic the system is.

I was asked to find the permutation of size 10 with maximum entropy. If you want to try and conceptualise what the entropy of a permutation is, imagine that you take all the interval maps that contain that a particular permutation somewhere inside them and then iterate each these maps repeatedly watching how 'complex' or 'messy' each map becomes. We define the entropy of that particular permutation to be the entropy of the least 'messy' map. More pragmatically, we calculate the entropy of a permutation by converting it to an interval map, then a 0-1 matrix and then finally computing the spectral radius of this matrix.

So what did I actually have to do? Well… my problem really had two parts. Firstly, I had to write a computer program that calculated the entropy of each size ten permutation and gave me the largest one, then secondly, I had to construct a mathematical proof that this permutation was indeed the maximal permutation.

Both these tasks were very interesting. I had done almost no programming before I started the project so trying to learn C in a few of days was quite challenging. Once I got the hang of it, however, I found it really useful. Computer programming saves a lot of arithmetic!

This may sound strange but another great part the vacation scholarship was being allowed to make mistakes. My supervisor initially gave me very little guidance on constructing the proof which really forced me to think for myself. Ultimately, this meant that I had to restart the proof many times but in the process I learnt a lot about how to approach problem solving.

Overall, I really enjoyed my time as a vacation scholar and would recommend it to anyone who is considering doing further study in mathematics.

Jonathan received an ICE-EM Vacation Scholarship in December 2005.
See www.ice-em.org.au/students.html#scholarships05