INTERNATIONAL CENTRE
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MATHEMATICS

**Symmetry Groups of Tropical Varieties**

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I did my project on tropical varieties with Max Flander as my partner and A/Prof John Groves as my supervisor. Tropical varieties have been around in different forms since the time of Newton, arising from the solutions of certain classes of differential equations. However since the 1970s, they are starting to be studied as algebraic objects in their own right.

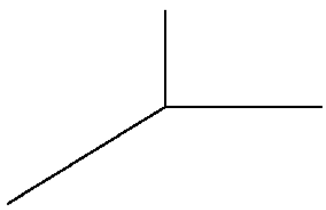
Tropical varieties arise naturally in the study of the so-called *tropical semiring*—basically a demented version of the real numbers, where addition and multiplication are redefined as

$$a + b \equiv \min(a,b)$$
$$a \cdot b \equiv a + b.$$

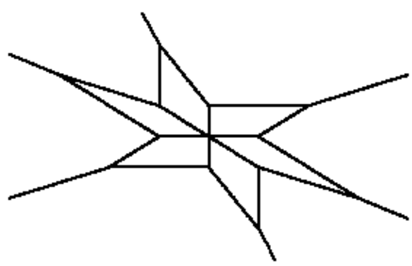
We can then use an ordinary polynomial to define a piece-wise linear function using these new definitions. For example, the function $f(x) = 0x^2 + 2x + 1$ would become
$f(x) = \min(2x, 2 + x, 1)$ (*)

The tropical variety of a polynomial is defined to be the set of all points for which its graph (defined tropically) is non-linear; equivalently, this is the set of points for which the minimum (*) occurs at least twice. From this definition, we can have tropical varieties in any dimension, but we focused on the 2- and 3-dimensional cases (since the 2D case is the only one where you can have pretty pictures!).

Our goal was to find out which groups could be realized as a symmetry group of a tropical variety. For example the variety for $x + y + 1$ (a “tropical line”) has symmetry group $D_3$ (the same symmetries as an equilateral triangle) as its symmetry group:



and the variety shown below has $C_6$ symmetry (same as a hexagon but with no reflections—can you see why?). We drew our pretty pictures using a program we wrote in Python and Mathematica.



**What we gained from our summer**

Working in a team, we had to learn skills such as how to split up the work effectively, and how to not annoy each other too much. The project opened my eyes to the mountains of research mathematicians still have to do.

Paul received an ICE-EM Vacation Scholarship in December 2005.
See [www.ice-em.org.au/students.html#scholarships05](www.ice-em.org.au/students.html#scholarships05)