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Enumeration of knotted graphs

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I worked with Maurice Chiodo and James Saunderson under the supervision of Craig Hodgson and Damian Heard on this vacation project. The aim of the project was to enumerate knotted graphs. Heard's PhD thesis involved writing a program to calculate the hyperbolic volume of knotted graph complements (among other things), so our enumeration was a useful test for this program. During the course of the project, in addition to the knotted graph enumeration, I learned about the mathematical foundations of the program, including hyperbolic geometry.

A knotted graph is a generalization of a knot or link, in that the objects being embedded in three-dimensional space are graphs, with vertices and edges, rather than a circle (for a knot) or a collection of at least two circles (for a link). Tables of certain types of knotted graphs, compiled by Litherland and Moriuchi, were already in existence. We decided to attempt to reproduce these tables, and in addition to enumerate all knotted graphs with up to four vertices, each vertex of degree three, and at most seven crossings.

Moriuchi had adapted Conway's method of insertion of algebraic tangles into basic polyhedra to the situation of knotted graphs, and we followed her in this. We wrote a program to perform this systematic enumeration, influenced heavily by the program plantri, which was written by Brendan MacKay (of ANU) and Gunnar Brinkmann.

This gave us a table of knotted graphs with many repetitions. To eliminate repetitions, we used Heard's program (in addition to some simple checks implemented in our program). This is usually the hard bit when one is enumerating knots or knotted graphs, and the excellent performance of Heard's program was the reason we were able to achieve an enumeration so much more ambitious than those previously attempted. We were able to enumerate all the knotted graphs that we aimed to.

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