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Continuous Time Random Walks and Fractional Calculus in Finance

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My project over the six weeks involved studying a general class of random walks via the Montroll-Weiss equation: $\hat{p}^*(k,s) = \dfrac{1-\psi^*(s)}{s(1-\hat{u}(k)\psi^*(s))}$. My first task was to read the original paper where it was published and try to derive the equation in a more formal manner. From that point I studied various papers where this equation was implemented. One paper which was particularly interesting was that of Mainardi et al which discussed a model for the survival probability for BOND futures. I also showed how fractional differential/integral operators arise naturally in this type of modelling. Fractional differential/integral operators such as:

1. $D_x^{-\alpha} f(x) = \dfrac{1}{\tilde{A}(\alpha)}\displaystyle\int_0^x (x-t)^{\alpha-1} f(t)dt$ (generalised iterated integration)

2. $\dfrac{d^{\acute{a}}}{dt^{\acute{a}}} f(t) = \dfrac{1}{\tilde{A}(1-\acute{a})}\displaystyle\int_0^t f'(\tau)\dfrac{d\hat{o}}{(t-\hat{o})^{\acute{a}}}, \text{ for } 0<\alpha<1$ (generalised derivative)

3. $\dfrac{d^{\beta}}{d^{\beta}|x|} f(x) = \tilde{A}(1+\beta)\dfrac{\sin(\frac{1}{2}\beta\pi)}{\pi}\displaystyle\int_0^\infty \dfrac{f(x+\hat{\imath})-2f(x)+f(x-\hat{\imath})}{\hat{\imath}^{1+\beta}}d\hat{\imath}$ (generalised derivative)

allow differential and integration to an arbitrary order and are not uniquely defined but rather are defined so that the operators have certain desirable properties, eg (2) preserves the algebraic form of the Laplace transform.

From here we looked at classifying a sub-class of CTRW from the Montrol-Weiss equation i.e. the diffusion and related processes by assuming some asymptotic behaviours of the waiting time and jump size probably density functions. Using this and the Caputo (2), Riesz (3) fractional derivatives we derived a fractional diffusion equation for this class of processes.

$$\frac{\partial^\alpha p(x,t)}{\partial t^\alpha} = \frac{\partial^\beta p(x,t)}{\partial |x|^\beta}, \quad \text{with} \quad p(x,0) = \delta(x)$$

Overall, the project was very illuminating and has had a significant impact on the academic path I will follow this year in my Honours. I am very greatly for the opportunity AMSI gave me.

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See www.ice-em.org.au/students.html#scholarships05