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Valuation of executive stock options

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An *option* gives the owner the *right* to buy or sell one unit of the underlying asset (e.g. stock) for a certain price at certain time(s) in the future. Under the Black-Scholes framework, the value of this derivative security, $V(x,t)$, satisfies the classical PDE

$$\frac{\partial V}{\partial t} = rV - rx\frac{\partial V}{\partial x} - \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V}{\partial x^2} \quad x>0,\ 0<t<T \text{ (expiry)}$$

subject to initial and boundary conditions depending on the payoff structure. Here x denotes asset price, r is the risk-free interest rate, and σ is the stock volatility.

We studied executive stock options (ESOs), a special class of derivative contracts. Due to their non-tradeable nature, there is no universal agreement on their pricing methodologies. The outcome of current research will have very practical significance for the implementation of new International Accounting Standards.

A substantial amount of effort was spent on understanding the mathematical foundations of the subject, namely stochastic calculus and measure theory. Wiener processes and Ito's Lemma were studied, and the notion of martingales was developed. The principle of static replication was used to price a range of exotic options, in the view that these securities have similar structures as the ESOs.

An extensive literature review was conducted on the recent development of ESOs, their incentive effects and valuation methods. We decided to adopt the binomial tree approach. Bermudan and compound options were priced using Matlab to test the convergence of our numerical schemes to analytic solutions in the case of multi-period contracts. The results agreed almost exactly with those calculated using the Black-Scholes model. A computational model based on a bivariate binomial tree was constructed to price a typical ESO contract with multi-vesting features and floating performance barriers. This project is currently under further development.

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See www.ice-em.org.au/students.html#scholarship.