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Coxeter groups

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In 1994 S. Ariki and K. Koike introduced the cyclotomic Hecke algebra $H_{n,r}$, which is a deformation of the group algebra of the wreath product of a cyclic group of order r by the symmetric group of degree n. More precisely, $H_{n,r}$ is defined by a presentation that reduces to a presentation of the wreath product when a certain parameter q is put equal to 1. By describing the representation theory of $H_{n,r}$, Ariki and Koike proved (amongst other things) that its dimension is $r^n n!$. Their proof of this result makes use of their description of the irreducible representations of the algebra, and, in particular, assumes that q is invertible. The aim of this project was to find a direct proof that the dimension is $r^n n!$ when q is invertible, and, if possible, find what happens when it is not.

The proof can be separated into two parts. The first part consists of describing a set of $r^n n!$ elements that span the algebra—this shows that the algebra can have dimension at most $r^n n!$. Then it must be shown that the spanning set is linearly independent. That is, we must show that these elements do not satisfy any 'hidden' linear relations that we might have discovered if we had been clever enough. So we must explicitly construct an algebra of dimension $r^n n!$ satisfying the defining relations.

It turns out that assuming that q is invertible makes a big difference, for without this assumption the algebra is no longer a free module of finite rank. In fact, it is spanned by the original $r^n n!$ elements, together with an infinite set of torsion elements.

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