INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

Elliptic curves and modular forms

Gareth White, School of Mathematics and Statistics, University of Sydney

During December 2005 and January 2006, I had the opportunity to undertake a vacation scholarship at Sydney University. Under the supervision of Dr David Kohel, I researched heights on abelian varieties, in particular focusing on the Mordell-Weil Theorem.

This theorem states that for an abelian variety defined over a number field k, the set of k-rational points is finitely generated as an abelian group. Since I had not heard of some of these terms before, I had to first grasp the language of abelian varieties before I could begin to understand this theorem and its proof.

An abelian variety can be thought of as a higher-dimensional generalisation of elliptic curves. If the number field is k, then this variety is defined over the algebraic closure of k. The k-rational points are those points in the variety whose coordinates all lie in k. The Mordell-Weil Theorem asserts that this set of points forms a finitely generated group.

The main reference books that I used throughout this project were "Diophantine Geometry: An Introduction" by Hindry and Silverman, and "Lectures on the Mordell-Weil Theorem" by Serre. I found this particular research topic very interesting because it uses many different areas of mathematics, such as algebra, analysis, geometry, and number theory.

At the end of my scholarship, I gave a presentation of my work as part of CSIRO's Big Day In event, where I was also able to observe projects conducted by other students at universities all around Australia. Overall, I found the scholarship to be a great experience, and the material that I learnt during the 6 weeks will certainly come in handy further down in my academic career.

Gareth received an ICE-EM Vacation Scholarship in December 2005.
See www.ice-em.org.au/students.html#scholarships05