ABSTRACTS: MAHLER LECTURER 2011: PETER SARNAK

Public Lectures:

Randomness in Number Theory

By way of concrete examples we discuss the dichotomy that in number theory the basic phenomena are either very structured or if not then they are random.The models forrandomness for different problems can be quite unexpected and understanding, and establishing the randomness is often the key issue. Conversely the fact that certain number-theoretic quantities behave randomly is a powerful source for the construction of much sought-after pseudo-random objects.

Number theory and the circle packings of Appolonius

Like many problems in number theory, the questions that arise from packing the plane with mutually tangent circles are easy to formulate but difficult to answer. We will explain the fundamental features of such packings and how modern tools from number theory, algebra and combinatorics are being used to answer some of these old questions.

Chaos, quantum mechanics and number theory

The correspondence principle in quantum mechanics is concerned with the relation between a mechanical system and its quantization. When the mechanical system are relatively orderly ("integrable"), then this relation is well understood. However when the system is chaotic much less is understood. The key features already appear and are well illustrated in the simplest systems which we will review. For chaotic systems defined number-theoretically, much more is understood and the basic problems are connected with central questions in number theory.

Colloquium Lectures:

Thin integer matrix groups and the affine sieve

Infinite index subgroups of integer matrix groups like SL(n,Z) which are Zariski dense in SL(n) arise in geometric diophantine problems (eg Integral Apollonian Packings) as well as monodromy groups associated with families of varieties. One of the key features needed when applying such groups to number theoretic problems is that the congruence graphs associated with these groups are "expanders".We will introduce and explain these ideas and review some recent developments especially those connected with the affine sieve.

Zeros and nodal lines of modular forms

One of the consequences of the recent proof by Holowinski and Soundararajan of the holomorphic "Quantum Unique Ergodicity Conjecture " is that the zeros of a classical holomorphic hecke cuspforms become equidistributed as the weight of the form goes to infinity. We review this as well as some finer features (first discovered numerically) concerning the locations of the zeros as well as of the nodal lines of the analogous Maass forms.The latter behave like ovals of random real projective plane curves, a topic of independent interest.

Mobius randomness and dynamics

The Mobius function mu(n) is minus one to the number of distinct prime factors of n if n has no square factors and zero otherwise. Understanding the randomness (often referred to as the "Mobius randomness principle") in this function is a fundamental and very difficult problem.We will explain a precise dynamical formulation of this randomness principle and report on recent advances in establishing it and its applications.

Horocycle flows at prime times

The distribution of individual orbits of unipotent flows in homogeneous spaces are well understood thanks to the work work of Marina Ratner. It is conjectured that this property is preserved on restricting the times from the integers to primes, this being important in the study of prime numbers as well as in such dynamics.We review progress in understanding this conjecture, starting with Dirichlet (a finite system), Vinogradov (rotation of a circle or torus), Green and Tao (translation on a nilmanifold) and Ubis and Sarnak (horocycle flows in the semisimple case).