ABSTRACTS: MAHLER LECTURER 2013 AKSHAY VENKATESH

Public lectures

How to stack oranges in three dimensions, 24 dimensions, and beyond

How can we pack balls as tightly as possible? In other words: to squeeze as many balls as possible into a limited space, what's the best way of arranging the balls? It's not hard to guess what the answer should be — but it's very hard to be sure that it really is the answer! I'll tell the interesting story of this problem, going back to the astronomer Kepler, and ending almost four hundred years later with Thomas Hales. I will then talk about stacking 24-dimensional oranges: what this means, how it relates to the Voyager spacecraft, and the many things we don't know beyond this.

Two centuries of prime numbers

Surprisingly, there have been fundamental new discoveries about prime numbers in the last decade, most recently by Yitang Zhang a few months ago. I'll survey some of our understanding of prime numbers in a nontechnical fashion, starting with the "music of the primes" — the strange oscillations between regions where primes are more common and more scarce — and concluding with a discussion of Zhang's discovery: prime numbers must occasionally come very close to one another.

Colloquium/specialist lectures

Dynamics and the geometry of numbers

It was understood by Minkowski that one could prove interesting results in number theory by considering the geometry of lattices in $\mathbb{R}^n$. (A lattice is simply a grid of points.) This technique is called the "geometry of numbers." We now understand much more about analysis and dynamics on the space of all lattices, and this has led to a deeper understanding of classical questions. I will review some of these ideas, with emphasis on the dynamical aspects.

The Cohen–Lenstra heuristics: from arithmetic to topology and back again

I will discuss some models of what a "random abelian group" is, and some conjectures (the Cohen–Lenstra heuristics of the title) about how they show up in number theory. I'll then discuss the function field setting and a proof of these heuristics, with Ellenberg and Westerland. The proof is an example of a link between analytic number theory and certain classes of results in algebraic topology ("homological stability").

Torsion in the homology of arithmetic groups

Take a Bianchi group — e.g, invertible 2×2 matrices with entries in the Gaussian integers — and abelianize it. The result is often a very large torsion group. I will discuss this phenomenon and how it relates to number theory.

From spherical harmonics to spherical varieties: harmonic analysis on homogeneous spaces and the Langlands program

Spherical harmonics are eigenfunctions of the Laplacian on the sphere; they are also closely related to understanding how the group of rotations decomposes the space of L2 functions on the sphere. We can ask similar questions replacing the sphere by other “highly symmetric” spaces — in particular, G/H where G and H are Lie groups. I will explain some of the work on this question and how it is related to the Langlands program.