ABSTRACTS: AMSI-CARMA LECTURER 2015
PROFESSOR JEREMY AVIGAD

Lecture Series Overview

Computers are changing the way we do mathematics, as well as introducing new research agendas. Computational methods in mathematics, including symbolic and numerical computation and simulation, are by now familiar. These lectures will explore the way that "formal methods," based on formal languages and logic, can contribute to mathematics as well.

In the 19th century, George Boole argued that if we take mathematics to be the science of calculation, then symbolic logic should be viewed as a branch of mathematics: just as number theory and analysis provide means to calculate with numbers, logic provides means to calculate with propositions. Computers are, indeed, good at calculating with propositions, and there are at least two ways that this can be mathematically useful: first, in the discovery of new proofs, and, second, in verifying the correctness of existing ones.

The first goal generally falls under the ambit of "automated theorem proving" and the second falls under the ambit of "interactive theorem proving." There is no sharp distinction between these two fields, however, and the line between them is becoming increasingly blurry. In these lectures, Professor Avigad provided an overview of both fields and the interactions between them, and speculated as to the roles they can play in mainstream mathematics.

Professor Avigad made the lectures accessible to a broad audience. The first lecture provided a self-contained overview. The remaining lectures were for the most part independent of one another, and did not rely on the first lecture.

Lecture 1
Formal Methods in Mathematics

In this lecture, Professor Avigad provided a general overview of automated and interactive theorem proving. He characterized the general projects and existing technology, and described some recent landmarks and successes in these fields. These included the verification of the Feit-Thompson theorem, the verification of the Kepler conjecture, the use of computers to establish results in algebraic topology, and the recent use of fast satisfiability solvers in connection with the Erdős discrepancy conjecture.

Lecture 2
Automated Theorem Proving

In this lecture, Professor Avigad explored automated methods in more detail, describing propositional theorem provers, first-order theorem provers, fast satisfiability methods, and the "satisfiability modulo theories" framework.

Lecture 3
Interactive Theorem Proving

In this lecture, Professor Avigad described the theory and technology behind contemporary proof assistants. He also discussed logical frameworks, proof languages, and the interaction with automated methods.

Lecture 4
Formal Methods in Analysis

In this lecture, Professor Avigad focused specifically on the use of formal methods in analysis. He discussed linear arithmetic, methods for real closed fields, and methods of handling more general nonlinear expressions, and explained how conventional numeric and symbolic approaches are integrated with formal methods.