CHEVALLEY GROUPS AND LIE ALGEBRAS WITH BUILT–IN STRUCTURE CONSTANTS

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1. Course abstract

The *structure constants* of a Lie algebra or Kac–Moody algebra $\mathfrak{g}$ are the constants that occur in the evaluation of the Lie bracket in terms of a choice of basis for the Lie algebra. In the known methods for computing them, structure constants are determined only up to a sign due to the existence of a canonical central extension of the root lattice of the Lie algebra by a cyclic group of order two. Determining a consistent system of signs of structure constants is a persistent problem in computational Lie theory, computational number theory and their applications.

For pairs of (real) roots α, β whose sum is a (real) root, the structure constant $n_{\alpha,\beta}$ given by $[x_\alpha, x_\beta] = n_{\alpha,\beta} x_{\alpha+\beta}$ can be determined in the rank 2 root subsystem generated by α and β. We give simple formulas for computing structure constants in all the possible rank 2 root subsystems that can occur. This gives a consistent system of signs over all root spaces for pairs of (real) roots whose sums are (real) roots and requires a classification of the root strings in the Lie algebra as well as a basis for which the structure constants are integral.

With this data, we can also construct algebraic groups as Chevalley groups using automorphisms of the Lie algebra and some additional external data such as a $\mathbb{Z}$–form of the universal enveloping algebra.

The basic structure theory of Lie algebras and Chevalley groups will be covered as needed. Some familiarity with finite dimensional Lie algebras is preferable though not required.

References

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