ACE Network Subject Information Guide

Asymptotic and perturbation methods for ordinary and partial differential equations

Semester 1, 2021

Administration and contact details

Host department	Mathematics and Statistics
Host institution	Macquarie University
Name of lecturer	Justin Tzou
Phone number	02 9850 8925
Email address	justin.tzou@mq.edu.au
Homepage	Click here to enter text.
Name of honours coordinator	Adam Sikora
Phone number	02 9850 8919
Email address	adam.sikora@mq.edu.au
Name of masters coordinator	Richard Garner
Phone number	02 9850 8923
Email address	richard.garner@mq.edu.au

Subject details

Handbook entry URL	https://unitguides.mq.edu.au/unit_offerings/129710/unit_guide
Subject homepage URL	tba
Honours student hand-out URL	tba
Start date:	Feb. 22, 2021
End date:	June 4, 2021
Contact hours per week:	2
Lecture day(s) and time(s):	Wednesday 10am-noon
Description of electronic access arrangements for students (for example, WebCT)	ILearn (details to be announced)

Subject content

1. **Subject content description**

Asymptotic and perturbation methods are powerful tools in applied mathematics research for obtaining approximate solutions to ordinary and partial differential equations in scenarios where exact solutions do not exist, and numerical solutions are deemed insufficiently informative or too time-consuming to compute. The aim of this course is to present students with the necessary tools to gain insight into and make predictions on the behaviour of differential equations pertaining to applications such as animal markings, vegetation patterning, search processes, and intracellular transport. Emphasis will be placed on how asymptotic methods are currently being used in these active areas of research. Topics covered include, asymptotic expansions, regular perturbation theory, eigenvalue perturbation theory, singular perturbation methods and boundary layer theory

2. Week-by-week topic overview

The following content will be distributed over the 12-week semester:

- **Introduction**
  - introduction to asymptotic series
  - perturbed algebraic equations

- **Regular perturbations**
  - eigenvalues of perturbed boundary value problems; solvability conditions
  - boundary value problems on perturbed domains
  - nonlinear initial value problems
  - stability analysis of solutions of parabolic PDEs

- **Singular perturbations**
  - matched asymptotic methods for boundary value problems
    - boundary layers
    - internal layers
    - nested layers
  - WKB method; turning points

- **Recent research and applications: asymptotic methods for PDEs**
  - weakly nonlinear analysis of parabolic PDEs with applications in
    - crime hotspot suppression
    - vortex dynamics in Bose-Einstein condensates
  - matched asymptotic methods for singularly perturbed parabolic PDEs in one spatial dimension
    - analysis of slow dynamics and stability of fully nonlinear localised patterns of reaction-diffusion systems

- hybrid asymptotic-numerical techniques for singularly perturbed parabolic and elliptic PDEs in two and three spatial dimensions
  - analysis of slow dynamics and stability of fully nonlinear localised spot patterns of reaction-diffusion systems
  - optimising the principal eigenvalue of the Laplacian on a punctured two-dimensional domain
  - narrow escape and narrow capture problems

3. Assumed prerequisite knowledge and capabilities

Differential equations (as obtained through, for example, a third year unit on differential equations; both ODEs and PDEs). Basic knowledge of the following may be beneficial:

- MATLAB (programming language)
- linear algebra
- numerical methods for differential equations

4. Learning outcomes and objectives

At the end of this course, students will be able to understand and apply asymptotic methods for analysing various classes of differential equations. Moreover, they will gain knowledge of the current state of research with regards to applications of asymptotic methods in certain areas of applied mathematics.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
K1	coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2	knowledge of research principles and methods
S1	cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2	cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3	cognitive skills to exercise critical thinking and judgement in developing new understanding
S4	technical skills to design and use in a research project
A2	to adapt knowledge and skills in diverse contexts

Learning Outcome Descriptors at AQF Level 8

Knowledge

K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines

K2: knowledge of research principles and methods

Skills

S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence

S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas

S3: cognitive skills to exercise critical thinking and judgement in developing new understanding

S4: technical skills to design and use in a research project

S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences

Application of Knowledge and Skills

A1: with initiative and judgement in professional practice and/or scholarship

A2: to adapt knowledge and skills in diverse contexts

A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters

A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

6.

Written lecture notes will be available for students and will be posted on the unit website. While not necessary, the following references and resources may be useful:

- *Advanced Mathematical Methods for Scientists and Engineers*, C.M. Bender and S.A. Orszag, McGraw-Hill (1978).
- *Asymptotic Analysis*, J.D. Murray, Spring-Verlag (1984).
- *Multiple Scale and Singular Perturbation Methods*, J. Kevorkian and J.D. Cole, Springer-Verlag (1996).
- *Introduction to Perturbation Methods*, M.H. Holmes, Springer-Verlag (1998).
- MATLAB (any version) or a free alternative such as GNU Octave

7. Assessment

Exam/assignment/classwork breakdown					
Exam		**Assignment**	100	**Class work**	0
Assignment due dates		Week 4	Week 8	Week 11	Week 13
Approximate exam date				N/A	

Institution Master of Research program details

Weight of subject in total honours assessment at host department	12.5% of BPhil
Thesis/subject split at host department	thesis is 90% of MRes
Honours grade ranges at host department:	
H1	85%
H2a	75%
H2b	65%
H3	50%