ACE Network Subject Information Guide

Elliptic partial differential equations from an elementary viewpoint

Semester 1, 2021

Administration and contact details

Host department	Department of Mathematics and Statistics
Host institution	University of Western Australia
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Subject details

Handbook entry URL	https://handbooks.uwa.edu.au/unitdetails?code=MATH4032
Subject homepage URL	
Honours student hand-out URL	
Start date:	22/02/21
End date:	21/05/21
Contact hours per week:	By appointment
Lecture day(s) and time(s):	The lectures are prerecorded and available on LMS
Description of electronic access arrangements for students (for example, WebCT)	Access to LMS through the University of Western Australia system

Subject content

1. Subject content description

Elementary introduction to the theory of second order partial differential equations of elliptic type. Development of the basic theory of harmonic functions and solutions of Laplace equations, with applications to some concrete problems in physics and engineering.

2. Week-by-week topic overview

Introductory Lecture 01: Introduction. Vibrating strings.

Introductory Lecture 02: Heat equation. Random walks.

Lecture 1: Laplacian and averages.

Lecture 2: Pizzetti's Formula (and basics of Gamma Function).

Lecture 3: Harmonic functions, Mean Value Theorem, Green's Identities.

Lecture 4: Semilinear equations, Pohozaev Identity, non-existence results.

Lecture 5: Laplace-Beltrami operator, tangential gradient and divergence, integration on tubular neighbourhoods.

Lecture 6: Tangential Divergence Theorem, Minkowski Integral Formula, Tangential Laplacian.

Lecture 7: Kelvin Transform.

Lecture 8: Fundamental Solution.

Lecture 9: Gravitational and electrostatic potentials. Physical interpretation of the Kelvin Transform.

Lecture 10: Weyl's Lemma. Navier-Stokes Equation.

Lecture 11: Maximum Principles.

Lecture 12: Maximum Principles on unbounded domains, the case of the halfspace.

Lecture 13: Green Function. Symmetry of Green Function.

Lecture 14: Poisson Kernel. Representation Formula. Poisson Kernel for balls and halfspaces.

Lecture 15: Analyticity of harmonic functions.

Lecture 16: Harnack Inequality.

Lecture 17: Harnack Convergence Theorem. Cauchy Estimates.

Lecture 18: Hopf Lemma.

Lecture 19: Liouville Theorem.

Lecture 20: Polynomial approximations.

Lecture 21: Spherical harmonics and eigenfunctions of the Laplace-Beltrami operator on the sphere.

Lecture 22: Orthogonality properties of spherical harmonics. Characterisation of the eigenfunctions of the Laplace-Beltrami operator on the sphere.

Lecture 23: Dimension of the vector space of homogeneous harmonic polynomials.

Lecture 24: Introduction to Legendre Polynomials.

Lecture 25: Uniqueness of spherical harmonics that are invariant under rotations fixing one axis. Weighted orthogonality properties of Legendre Polynomials.

Lecture 26: Useful identities for Legendre Polynomials.

Lecture 27: An application to physical geodesy: determination of the gravitational potential from the available experimental measurements of the gravity anomaly.

Lecture 28: Rigidity of bars and plates. Almansi's Representation Formula for polyharmonic functions.

Lecture 29: Almansi's solution for the hinged plate.

Lecture 30: The Soap Bubble Theorem.

Lecture 31: The Isoperimetric Problem.

Lecture 32: Straight bars subject to torsion and viscous fluids in straight pipes: an overdetermined problem posed by R. Fosdick.

Lecture 33: Serrin's Overdetermined Problem. Torsional rigidity and Saint-Venant's Problem. Hadamard Variational Formula. Optimal heating problems.

Lecture 34: Conclusions.

3. Assumed prerequisite knowledge and capabilities

To maximise the learning experience, it may be convenient to review some of the basic material on mathematical analysis, such as: Fundamental Theorem of Calculus, Multivariate Calculus, Chain Rule, Taylor Formula, Polar Coordinates, Change of Variables, Divergence Theorem, Convolutions, Implicit Function Theorem, Inverse Function Theorem, Fubini-Tonelli's Theorem, Integration under the Integral Sign, etc.

In any case, these are handy tools that are indispensable for all mathematicians and their use pops up in the everyday life of all researchers, so it is advisable to know them well, independently of this specific unit.

One of the benefits of this unit however is also to help the participants consolidate their skills on these essential techniques from mathematical analysis and to make them confident of exploiting these devices successfully and creatively.

4. Learning outcomes and objectives

Being exposed to an elementary introduction to the theory of second order partial differential equations of elliptic type. Familiarising with the basic theory of harmonic functions and solutions of Laplace equations, with applications to some concrete problems in physics and engineering. Reviewing and consolidating the knowledge of analysis and multivariate calculus.

5. Learning resources

Serena Dipierro, Enrico Valdinoci, *Elliptic partial differential equations from an elementary viewpoint*, https://arxiv.org/pdf/2101.07941.pdf

6. Assessment

Each lecture contains some **exercises**. These exercises are the students' assignments and they contribute to the final mark. Some exercise is given with some hint, and most of the exercises are solved, one way or another, in the notes of the course. Students can follow the hints for doing the exercise or/and take advantage of the notes. Students can also neglect the hints and solve the exercise by independent strategies. In any case, the solution of the exercises must be clear, self-contained and exhaustive. When the students submit their assignments, are required to write the exercises in order and label them (labeling exercise n in lecture m as EX-n-LE-m).

Students taking this unit for credit must also submit a 15 minute (or longer, according to students' preferences) **video** with a mathematical **presentation**. The topic of the presentation has to be settled with the instructors. Possible topics of the presentation are: Arzelà–Ascoli Theorem, Fréchet–Kolmogorov Theorem, Maximum Modulus Principle, Dominated Convergence Theorem, Vitali Convergence Theorem, Stone-Weierstrass Theorem, etc.

Students who do not feel comfortable by submitting the video presentation may request an alternative form of evaluation.

DEADLINES:

Friday 19 March, 6 PM Perth time: submit the first 20 exercises;

Friday 23 April, 6 PM Perth time: submit the subsequent 12 exercises;

Friday 21 May, 6 PM Perth time: submit the last 14 exercises.

The oral presentation can be submitted by a video by LMS anytime before Friday 11 June, 6 PM Perth time.