Lagrangian and Hamiltonian Dynamics: Background Skills Quiz

This quiz is for your benefit, I will not collect or mark it. It is to give you an idea of the type of skills that will be needed to be able to cope with this course. The material here is based on High School, 1st Year and 2nd year maths courses. You should be able to answer the questions without undue effort within the space provided.

Differential Equations

Give the general solution of each of the following ODEs

$\frac{d^2y}{dx^2} + 9y = 0$	
$\frac{d^2y}{dx^2} - 9y = 0$	
$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$	
$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$	
$\frac{d^2y}{dx^2} + 9y = 7 + 2x$	
$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$	
$\frac{d^2y}{dx^2} + 9y = \sin(3x)$	

Change of variables

Rewrite the following expressions in polar coordinates using $x = r\cos\phi$ and $y = r\sin\phi$.

$\int xdy - ydx$	
$\iint x\, dxdy$	
$dx^2 + dy^2$	

Derivatives

Calculate $\frac{dy}{dx}$ for each of the following curves.
You may write your answer in terms of the most convenient set of variables.

$y = \sin^{-1}(x)$	
$\cos(x) = \ln(y) + \sinh(y)$	
$x = \sin(t) - t; \quad y = \tan(t)$	
$F(x, y) = 0$	

Integrals

$\int x \ln(x)\, dx$	
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	See if you can find two different substitutions

Linear algebra

For each of the following write down as many **relevant** properties about the square matrix A, or its eigenvalues λ or eigenvectors as you can remember

$A\,\underline{x} = \underline{b}$ has a unique solution for $\underline{x}$	
$A\,\underline{x} = 0$ has a non-zero solution for $\underline{x}$	
A is positive definite	
A is real symmetric	

Vector algebra and Geometric Interpretation

Describe geometrically the set of points $\mathbf{r}$ that satisfy the following conditions where c is a constant and $\mathbf{n}$ is a constant unit vector.

$\mathbf{r} \cdot \mathbf{n} = 0$	
$\mathbf{r} \cdot \mathbf{n} = c$	
$\mathbf{r} \times \mathbf{n} = \mathbf{0}$	

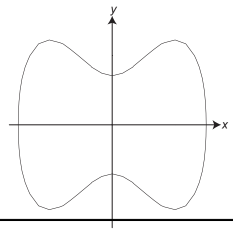
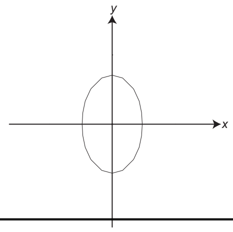
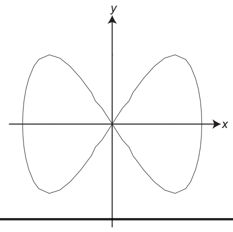
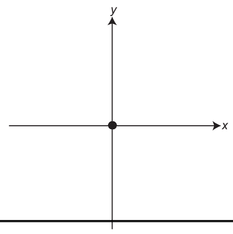
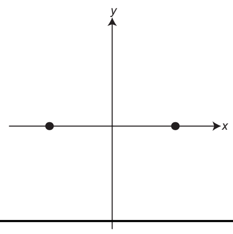
Trigonometry

Simplify the following expressions

$\dfrac{1 - \cos(2\theta)}{\sin(2\theta)}$	
$\sin^{-1}(x) + \cos^{-1}(x)$	
$\tan(\sin^{-1}(x))$	
$\cosh^2 x + \sinh^2 x$	

Curve sketching and logical thinking

Consider the family of curves given by the real equation $y^2 + x^4 + 2ax^2 + b = 0$. Match the given conditions on the left to the correct curves on the right and sketch the missing curve. Each condition matches exactly one curve.

(1) $a < 0$ and $b < 0$		
(2) $a < 0$ and $b = 0$		
(3) $a < 0$ and $0 < b < a^2$		
(4) $a < 0$ and $b = a^2$		
(5) $a \geq 0$ and $b < 0$		
(6) $a \geq 0$ and $b = 0$		

What is the condition for there to be no real points on the curve?

Blank Page for Notes and Working

Skills and Topics (I should brush up on)	I don't even remember learning this	I should revise this	Yeah, I know this!
Homogeneous 2nd order Linear ODEs			
Inhomogeneous 2nd order Linear ODEs			
ODEs with repeated roots (or resonant RHS)			
Manipulating differentials (dx, dy etc.)			
Change of variables in integral			
Change of variables in double integral			
Differentiating trig functions			
Differentiating inverse trig functions			
Implicit differentiation			
Integration by parts			
Integration by substitution			
Integration by partial fractions			
Properties of determinants			
Properties of eigenvalues			
Properties of eigenvectors			
Scalar and vector products			
Trig identities			
Inverse Trig functions			
Hyperbolic Trig functions			
Quadratic equations and discriminants			

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SOLUTIONS

This quiz is for your benefit, I will not collect or mark it. It is to give you an idea of the type of skills that will be needed to be able to cope with this course. The material here is based on High School, 1st Year and 2nd year maths courses. You should be able to answer the questions without undue effort.

Differential Equations

Give the general solution of each of the following ODEs

$\frac{d^2y}{dx^2} + 9y = 0$	$y(x) = A\sin(3x) + B\cos(3x)$
$\frac{d^2y}{dx^2} - 9y = 0$	$y(x) = A\exp(3x) + B\exp(-3x)$
$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$	$y(x) = A\exp(-2x) + B\exp(-3x)$
$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$	$y(x) = A\exp(2x) + B\exp(3x)$
$\frac{d^2y}{dx^2} + 9y = 7 + 2x$	$y(x) = A\sin(3x) + B\cos(3x) + (7 + 2x)/9$
$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$	$y(x) = A\exp(-2x) + Bx\exp(-2x)$
$\frac{d^2y}{dx^2} + 9y = \sin(3x)$	$y(x) = A\sin(3x) + B\cos(3x) + y_p(x)$ $y_p(x) = (\sin(3x) - 6x\cos(3x))/36$

Change of variables

Rewrite the following expressions in polar coordinates using $x = r\cos\phi$ and $y = r\sin\phi$.

$\int xdy - ydx$	$= \int r^2 d\phi$
$\iint x\,dxdy$	$= \iint r\cos\phi\, rdrd\phi$
$dx^2 + dy^2$	$= dr^2 + r^2 d\phi^2$

Derivatives

Calculate $\frac{dy}{dx}$ for each of the following curves.
You may write your answer in terms of the most convenient set of variables.

$y = \sin^{-1}(x)$	$y' = \frac{1}{\sqrt{1-x^2}}$
$\cos(x) = \ln(y) + \sinh(y)$	$y'(x) = -\sin(x)(y^{-1} + \cosh y)^{-1}$
$x = \sin(t) - t; \quad y = \tan(t)$	$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{(\sec t)^2}{\cos t - 1}$
$F(x, y) = 0$	$\frac{dy}{dx} = -\frac{\partial_x F}{\partial_y F}$

Integrals

$\int x \ln(x)\, dx$	$= \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \cosh^{-1}(x/a) + C$ using $x = a \cosh t$ $= \log(x/a + \sqrt{(x/a)^2 - 1}) + C$ using $x = a \sec t$

Linear algebra

For each of the following write down as many **relevant** properties about the square matrix A, or its eigenvalues λ or eigenvectors as you can remember

$A\,\underline{x} = \underline{b}$ has a unique solution for $\underline{x}$	A invertible, all $\lambda \neq 0$
$A\,\underline{x} = 0$ has a non-zero solution for $\underline{x}$	$\lambda = 0$, $\det A = 0$, A not invertible
A is positive definite	all $\lambda > 0$, $\underline{x}^t A \underline{x} > 0$ for any $\underline{x}$
A is real symmetric	all λ real, A is diagonalisable by orthogonal transformation

Vector algebra and Geometric Interpretation

Describe geometrically the set of points $\mathbf{r}$ that satisfy the following conditions where c is a constant and $\mathbf{n}$ is a constant unit vector.

$\mathbf{r} \cdot \mathbf{n} = 0$	plane through origin orthogonal to $\mathbf{n}$
$\mathbf{r} \cdot \mathbf{n} = c$	plane orthogonal to $\mathbf{n}$, distance $\lvert c \rvert$ to origin
$\mathbf{r} \times \mathbf{n} = \mathbf{0}$	line through origin parallel to $\mathbf{n}$

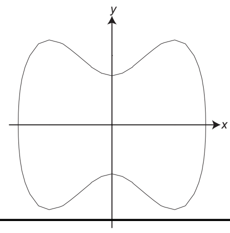
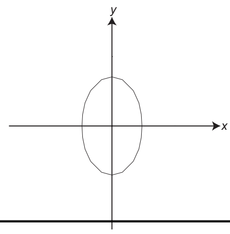
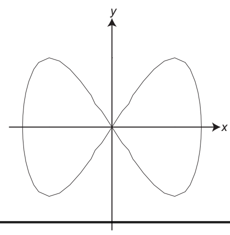
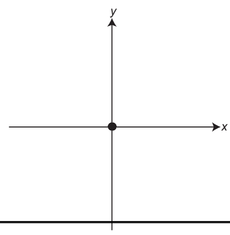
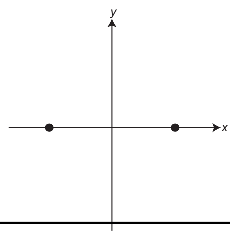
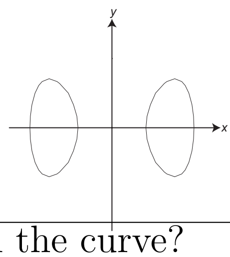
Trigonometry

Simplify the following expressions

$\dfrac{1 - \cos(2\theta)}{\sin(2\theta)}$	$= \tan\theta$
$\sin^{-1}(x) + \cos^{-1}(x)$	$= \frac{\pi}{2}$
$\tan(\sin^{-1}(x))$	$= \frac{x}{\sqrt{1-x^2}}$
$\cosh^2 x + \sinh^2 x$	$= \cosh(2x)$

Curve sketching and logical thinking

Consider the family of curves given by the real equation $y^2 + x^4 + 2ax^2 + b = 0$. Match the given conditions on the left to the correct curves on the right and sketch the missing curve. Each condition matches exactly one curve.

(1) $a < 0$ and $b < 0$	(1)	
(2) $a < 0$ and $b = 0$	(5)	
(3) $a < 0$ and $0 < b < a^2$	(2)	
(4) $a < 0$ and $b = a^2$	(6)	
(5) $a \geq 0$ and $b < 0$	(4)	
(6) $a \geq 0$ and $b = 0$	(3)	

What is the condition for there to be no real points on the curve?

$a < 0$ and $b > a^2$