Optimisation for deep learning: Pre-quiz

1 Linear Algebra

Exercise 1

given the matrices

$$A = \begin{bmatrix} 4 & 5 & 2 \\ 2 & 2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 5 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$$

Find (or explain why this is not possible):

1. AB
2. $A + B$
3. $A^T B$
4. $\det(A)$
5. A^{-1}

Exercise 2

Given the augmented matrix:

$$A = \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 1 & 1 & 2 \\ 2 & 2 & -1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

1. Use Gaussian elimination to reduce A to row echelon form.
2. Give the solution to the corresponding system of equations. Write your answer in vector form.

2 Calculus

Exercise 3

Calculate the first partial derivatives of the function:

$$f(x, y) = x^2 y + y^2 e^{xy}$$

Solutions to the Exercises

Solution 1

1. $\begin{bmatrix} 28 & 31 \\ 12 & 12 \\ -10 & -16 \end{bmatrix}$
2. Not possible - the matrices do not have the same size.
3. 12
4. $\begin{bmatrix} -\frac{1}{3} & \frac{5}{6} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{5}{6} & -\frac{1}{6} \end{bmatrix}$

Solution 2

1. $$A = \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 1 & 1 & 2 \\ 0 & -2 & 1 & -2 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$
2. $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ \frac{3}{2} \\ 0 \\ 1 \\ 1 \end{bmatrix} x_5$$

Solution 3

$$\frac{\partial f}{\partial x} = 2xy + y^3 e^{xy}$$
$$\frac{\partial f}{\partial y} = x^2 + 2y e^{xy} + xy^2 e^{xy}$$