

ACE Network Subject Information Guide

MATH4313: Functional Analysis

Semester 1, 2022

Administration and contact details

| Host department | School of Mathematics and Statistics | |
|-----------------------------|---|--|
| Host institution | The University of Sydney | |
| | | |
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Subject details

| Handbook entry URL | https://www.sydney.edu.au/units/MATH4313 |
|---|---|
| Subject homepage URL | https://www.maths.usyd.edu.au/u/UG/HM/MATH4313/ |
| Honours student hand-out URL | https://canvas.sydney.edu.au/courses/31411 |
| | |
| Start date: | 1 March 2022 |
| End date: | 3 June 2022 |
| Contact hours per week: | 3 lectures and 1 tutorial (13 Weeks) |
| Lecture day(s) and time(s): | Lectures Mon 09:00 – 11:00 and Wed 09:00 – 10:00, |
| | Lectures will be recorded and available through Canvas. |
| | Tutorial Wed, 10:00 – 11:00 |
| | |
| Description of electronic access arrangements | Access to Canvas to be arranged |
| for students (for example, WebCT) | |

Subject content

1. Subject content description

A A C E

Functional analysis is one of the major areas of modern mathematics. It can be thought of as an infinite-dimensional generalisation of linear algebra and involves the study of various properties of linear continuous transformations on normed infinite-dimensional spaces. In this unit you will start with a review of the topology of metric spaces, and then move on to cover topics that include normed vector spaces, Banach and Hilbert spaces; bounded and closed linear operators; abstract Fourier series; uniform boundedness, closed graph and the open mapping theorem; dual spaces and the Hahn-Banach theorem as well as spectral theory. If time permits the unit will cover the spectral theory of compact operators. While the unit constitutes a self-contained introduction to functional analysis, we will also be looking at some more concrete applications, particularly in tutorials.

2. Week-by-week topic overview

Tentative Schedule:

- Week 1: Review of the topology of metric spaces, compactness and continuity
- Week 2: Normed linear spaces and basic properties of linear operators
- Week 3: Finite and infinite dimensional Banach spaces
- Week 4: Banach algebras and the Stone-Weierstrass Theorem
- Week 5: Hilbert spaces, orthonormal systems, projection theorem, abstract Fourier series
- Week 6: Baire's theorem and the open mapping theorem
- Week 7: The closed graph theorem, the uniform boundedness theorem
- Week 8: Closed operators
- Week 9: Duality and the Hahn-Banach theorem
- Week 10: Weak convergence
- Week 11: Duality in Hilbert spaces and the Lax-Milgram Theorem
- Week 12: Basics of spectral theory, ascent and descent of linear operators
- Week 13: The spectrum of compact operators

3. Assumed prerequisite knowledge and capabilities

Working knowledge in abstract linear algebra and analysis, including working with inner products on real or complex vector spaces, the Euclidean norm, supremum and infimum and basic point set topology. Some knowledge of metric spaces and Lebesgue integration is an advantage but not required.

4. Learning outcomes and objectives

See https://www.sydney.edu.au/units/MATH4313 (follow the link to Semester 1, 2022)

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

A A C E

| AQF Program Learning Outcomes addressed in | Associated AQF Learning Outcome Descriptors |
|--|---|
| this subject | for this subject |
| Insert Program Learning Outcome here | Choose from list below |
| Insert Program Learning Outcome here | Choose from list below |
| Insert Program Learning Outcome here | Choose from list below |
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Learning Outcome Descriptors at AQF Level 8 Knowledge

K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines

K2: knowledge of research principles and methods

Skills

S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence

S2: cognitive and technical skills to demonstrate a broad understanding of a body of

knowledge and theoretical concepts with advanced understanding in some areas

S3: cognitive skills to exercise critical thinking and judgement in developing new understanding

S4: technical skills to design and use in a research project

S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences

Application of Knowledge and Skills

A1: with initiative and judgement in professional practice and/or scholarship

A2: to adapt knowledge and skills in diverse contexts

A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters

A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

Daniel Daners, Introduction to Functional Analysis (Lecture Notes, University of Sydney, 2021, will be made available as a PDF).

6. Assessment

| Exam/assignment/classwork breakdown | | | | | | | |
|---|------|---------------|---------------|------------------------|--|--|--|
| Exam | 70 % | Assignment | 3 x10% | Class work | | | |
| | | | | | | | |
| Assignment due dates | | 23 March 2022 | 29 April 2022 | 27 May 2022 | | | |
| See also https://www.sydney.edu.au/units/MATH4313 (follow the link to Semester 1, 2022) | | | | | | | |
| Approximate exam date | | | | Exam period 13-24 June | | | |

Institution honours program details

σ Σ N E T W O R K

| Weight of subject in total honours assessment at | One out of four coursework units |
|--|----------------------------------|
| host department | |
| Thesis/subject split at host department | 50% + 50% |
| Honours grade ranges at host department | |
| H1 | 80 - 100% |
| H2a | 75 – 79% |
| H2b | 70 – 74% |
| Н3 | 65 – 69% |

Institution masters program details

| Weight of subject in total masters assessment at host department | This is one of 6 coursework units in the 2-year masters of mathematical science program |
|--|---|
| Thesis/subject split at host department | 2/3 coursework and 1/3 thesis (2-year program) |
| Masters grade ranges at host department | |
| H1 | Enter range % |
| H2a | Enter range % |
| H2b | Enter range % |
| Н3 | Enter range % |