ACE Network Subject Information Guide

MATH4061: Metric Spaces

Semester 1, 2022

Administration and contact details

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Subject details

Handbook entry URL	https://www.sydney.edu.au/courses/units-of-study/2022/math/math4061.html
Subject homepage URL	https://www.sydney.edu.au/units/MATH4061/2022-S1C-ND-CC
Honours student hand-out URL	N/A
Start date:	21/02/2022
End date:	27/05/2022 (Exam period 06-18/06/2022)
Contact hours per week:	3 hours of lectures and 1 hour of tutorials
ACE enrolment closure date:	04/03/2022
Lecture day(s) and time(s):	Lectures: Tue 9-11, Wed 11-12. Tutorial: Thu 12-13:00
Description of electronic access arrangements for students (for example, WebCT)	Access to the learning management system (Canvas) will be arranged.

Subject content

1. **Subject content description**

Topology, developed at the end of the 19th Century to investigate the subtle interaction of analysis and geometry, is now one of the basic disciplines of mathematics. A working knowledge of the language and concepts of topology is essential in fields as diverse as algebraic number theory and non-linear analysis. This unit develops the basic ideas of topology using the example of metric spaces to illustrate and motivate the general theory. Topics covered include: Metric spaces, convergence, completeness and the Contraction Mapping Theorem; Metric topology, open and closed subsets; Topological spaces, subspaces, product spaces; Continuous mappings and homeomorphisms; Compactness Connectedness Hausdorff spaces and normal spaces. You will learn methods and techniques of proving basic theorems in point-set topology and apply them to other areas of mathematics including basic Hilbert space theory and abstract Fourier series. By doing this unit you will develop solid foundations in the more formal aspects of topology, including knowledge of abstract concepts and how to apply them. Applications include the use of the Contraction Mapping Theorem to solve integral and differential equations.

2. Week-by-week topic overview

Week 1: Definition and basic examples of metric spaces including normed vector space. Limits and continuity, the topology of metric spaces.

Week 2: Topological notions: closed sets, interior, closure, boundary, derived set. Simple examples and common constructions of topologies. Topologically equivalent metrics.

Week 3: Convergence of sequences, sequential characterisations of closed sets in metric spaces, local bases, first countable topological spaces, uniqueness of limits and introduction to separation axioms. Sequential characterisation of continuity.

Week 4: Cauchy sequences and the completeness of metric spaces, the completion of a metric space. Topological versus metric properties.

Week 5: Compact topological spaces, properties of continuous functions on compact sets. Characterisations of compact metric spaces.

Week 6: More on compactness. Separable, second countable and Lindelöf spaces.

Week 7: Uniform continuity, extension of uniformly continuous functions on dense subsets.

Week 8: The contraction mapping theorem and applications: Existence and uniqueness of solutions to ordinary differential equations, inverse and implicit function theorems.

Week 9: Connected topological spaces, connected components, continuous functions on connected sets.

Week 10: Normal spaces, Urysohn's Lemma and the Tieze Extension Theorem.

Week 11: Baire's theorem and applications

Week 12: Hilbert spaces, orthogonal projections and abstract Fourier series.

Week 13: More on Hilbert space theory, revision.

3. Assumed prerequisite knowledge and capabilities

Real (and ideally complex) analysis, multivariable calculus, abstract linear algebra. An ability to grasp abstract concepts and proofs.

4. Learning outcomes and objectives

See https://www.sydney.edu.au/units/MATH4061/2022-S1C-ND-CC

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below

Learning Outcome Descriptors at AQF Level 8

Knowledge

K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines

K2: knowledge of research principles and methods

Skills

S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence

S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas

S3: cognitive skills to exercise critical thinking and judgement in developing new understanding

S4: technical skills to design and use in a research project

S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences

Application of Knowledge and Skills

A1: with initiative and judgement in professional practice and/or scholarship

A2: to adapt knowledge and skills in diverse contexts

5. Learning resources

All resources will be provided through the learning management system (Canvas)

6. Assessment

Exam/assignment/classwork breakdown	
See https://www.sydney.edu.au/units/MATH4061/2022-S1C-ND-CC	
Approximate exam date	Exam period 06-18/06/2022

Institution honours program details

Weight of subject in total honours assessment at host department	25% of coursework component (Standard 6 credit point unit, 25% of full time student load for one semester)
Thesis/subject split at host department	50% coursework, 50% thesis
Honours grade ranges at host department	
H1	Enter range %
H2a	Enter range %
H2b	Enter range %
H3	Enter range %

Institution masters program details

Weight of subject in total masters assessment at host department	Click here to enter text.
Thesis/subject split at host department	Click here to enter text.
Masters grade ranges at host department	
H1	Enter range %
H2a	Enter range %
H2b	Enter range %
H3	Enter range %