ACE Network Subject Information Guide

Methods and Theory of Modern Optimisation

Semester 1, 2022

Administration and contact details

Host department	Mathematical Sciences, School of Science
Host institution	RMIT University
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Subject details

Handbook entry URL	NA
Subject homepage URL	https://www.mocao.org/ACE2022/
Honours student hand-out URL	https://www.mocao.org/ACE2022/
Start date:	28th February
End date:	27th May
Contact hours per week:	2 hours lectures + 1 hour online consultation
Last day for ACE enrolment:	18th March
Lecture day(s) and time(s):	Tuesdays 2-4pm AEDT
Description of electronic access arrangements for students (for example, WebCT)	https://www.mocao.org/ACE2022/

Subject content

- **Subject content description**

Nonlinear and convex optimisation has gained wide use in data mining and machine learning in recent years. This has spawn new research and renewed interest in the underlying mathematical foundations behind descent methods. The main tool of machine learning is stochastic gradient descent which is corresponds to a statistically unbiased approximation of a deterministic descent method and is used widely in regression and classification problems. Thus, a renewed interest in first order gradient-based convex optimisation has arisen out of the study of this and similar problems. The nonsmooth version arises in other problems found in Machine learning, such as the regularised risk minimisation with binary hinge loss. In the area of signal processing, similar nonsmooth problems are faced when recovering corrupted signals with sparse support (compressed sensing). Moreover, optimisation under uncertainty also draws its techniques from the same well, with methods used to solve stochastic optimisation problems exploiting decomposition methods to create parallel programming approaches based on constrained convex optimisation, such as the so-called alternating direction method of multipliers (ADMM). Other feasibility methods based on projection-based methods also can be viewed as arising out of special cases of these general methods.

In the course we will take a modern view of the study of these and related problems, and the algorithms used to solve them. You will be introduced to the language of convex and nonsmooth optimisation, and shown how this powerful mathematical machinery allows one to analyse optimisation problems and develop algorithms for the solution of these problems. We will study the application of ADMM to various problem sets, including stochastic optimisation and feasibility problems. The ability to apply these techniques within the hyperspace of symmetric matrices allows the same methods to be applicable to a wider class of problems, and this aspect will also be explored.

Ultimately, we will look at the use of these techniques to some of the areas of application discussed above. Depending on the background of the student, scope is available in this course for projects involving theoretical work and/or more practical work involving the coding of methods.

- **Week-by-week topic overview**

Week 1: Convexity Preserving Operations, inner product spaces, symmetric matrices and Frobenius norm.
Week 2: Supports, Separation, relative interior and Subdifferentiability
Week 3: Fenchel Conjugate, Fenchel duality and Application in Optimization
Week 4: Lagrangian Duality and Penalty methods
Week 5: Introduction to Positive Semi-Definite problems.
Week 6: Simple subgradient methods, step size for methods and convergence
Week 7: Using subgradient methods in LPs and LR. Improvements when we have a gradient.
Week 8: The method of multipliers and alternating direction method of multipliers (ADMM).
Week 9: Using ADMM: Linear and Quadratic Programming Convex optimisation and Alternating Projections.
Week 10: Stochastic optimisation. Feasibility problems i.e. matrix completion
Week 11: Applications in Machine learning

Week 12: Inverse problems and Compressed sensing

- **Assumed prerequisite knowledge and capabilities**

  Necessary: Any under-graduate courses in real analysis, basic vector calculus and linear algebra.

  Desirable: Familiarity with a computer programming language like Matlab, Julia, Python, etc.

- **Learning outcomes and objectives**

  On completion of this course student should have gained:

  - Knowledge of advanced mathematical techniques in convex optimisation;
  - The ability to reformulate problems and identify and/or develop numerical algorithms for the solution of these problems;
  - Exposure to important classes of real world problems in areas like machine learning and signal processing.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	**Associated AQF Learning Outcome Descriptors for this subject**
Knowledge of advanced mathematical techniques in convex optimisation	K1 (knowledge)
The ability to reformulate problems and identify and/or develop numerical algorithms for the solution of these problems	S1 (Skills)
Exposure to important classes of real work problems in areas like machine learning and signal processing	S2 (Skills)
Insert Program Learning Outcome here	Choose from list below

- **Learning resources**

Students will be provided with a full set of lecture notes will be provided that are typeset in Latex. Preferred computing platforms include Matlab, Julia and Python.

- **Assessment**

Exam/assignment/classwork breakdown					
Exam/end of semester project	30%	**Mid-semester Assignment**	30%	**Weekly Assessments**	40%
Mid-semester Assignment due dates		Week 6			
Approximate exam date				Week 12-14	

Institution honours program details

Weight of subject in total honours assessment at host department	12.5%
Thesis/subject split at host department	25%thesis / 75% course work
Honours grade ranges at host department	
H1	80 - 100%
H2a	75 - 79%
H2b	70 - 79 %
H3	60 - 79%