ACE Network Subject Information Guide

Topological Groups

Semester 2, 2022

Administration and contact details

Host department	School of Mathematical and Physical Sciences
Host institution	University of Newcastle
Name of lecturer	George Willis
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Subject details

Handbook entry URL	
Subject homepage URL	
Honours student hand-out URL	
Start date:	July 18th, 2022
End date:	October 28th, 2022
Contact hours per week:	3
Census date:	August 12th, 2022
Lecture day(s) and time(s):	**Tuesday 11am – 1pm and Tuesday 3-4pm AEST** 11am-12pm Lecture 12pm-1pm Q&A and problem session 3-4pm Lecture
Description of electronic access arrangements for students (for example, WebCT)	Click here to enter text.

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Subject content

1. Subject content description

Topological groups combine continuity and symmetry. The focus of this course is on locally compact groups, which are the exact setting to which the methods of differentiation, integration and linear algebra seen in 1st-year mathematics extend. These extended methods then apply to such fields as mathematical physics, harmonic analysis and number theory. The course thus offers a deeper perspective on key topics in undergraduate mathematics and provides a foundation for further work in several fields.

The course develops the theory of locally compact groups to the point where the solution of Hilbert's Fifth problem (a major milestone of 20th century mathematics) and the existence of the Haar integral can be understood. These theorems will be illustrated with examples and proved in special cases. (Complete proofs would take another course.) The latter part of the course introduces recent introduces much more recent ideas on totally disconnected, or 0-dimensional, groups.

2. Week-by-week topic overview

1. Definition of topological groups. Examples.
2. Basic structure of topological groups. Locally compact groups and fields.
3. Continuous functions and uniform continuity. Integrals on function spaces.
4. Translation-invariant (Haar) integrals exists and are unique. Examples.
5. The modular function. More examples.
6. Matrix groups are examples of Lie groups. Projective limits. Profinite groups.
7. Hilbert's Fifth Problem. Approximation by Lie groups. Examples.
8. Approximation of compact groups by matrix groups.
9. Totally disconnected, locally compact (t.d.l.c.) groups. Van Dantzig's Theorem. Examples.
10. The scale function and minimising subgroups.
11. Tidy subgroups, characterisation of minimising subgroups. Proofs and examples.
12. Either a) proof of existence of the Haar integral on t.d.l.c. groups, or b) flat groups of automorphisms of a t.d.l.c. group.

3. Assumed prerequisite knowledge and capabilities

Topology: compactness, connectedness, the product topology.

Algebra: groups and fields, linear algebra.

Analysis: Riemann integration.

Discrete mathematics: graphs, modular arithmetic.

Students are expected to transfer their current knowledge to new contexts, work independently on exercises, write clear solutions, and follow logical arguments.

4. Learning outcomes and objectives

- Advanced knowledge and skills in topological algebra.
- View of higher level connections between elementary topics in the undergraduate program.
- Foundation for further work in functional analysis, algebra, mathematical physics, or number theory.
- Experience with writing proofs.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	**Associated AQF Learning Outcome Descriptors for this subject**
Advanced knowledge and skills in topological algebra	K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
View of higher level connections between elementary topics in the undergraduate program	S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
Advanced knowledge and skills in topological algebra	S4: technical skills to design and use in a research project
Foundation for further work in functional analysis, algebra, mathematical physics, or number theory	S4: technical skills to design and use in a research project
Experience with writing proofs	S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Foundation for further work in functional analysis, algebra, mathematical physics, or number theory	A2: to adapt knowledge and skills in diverse contexts

Learning Outcome Descriptors at AQF Level 8 **Knowledge** K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines K2: knowledge of research principles and methods **Skills** S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas S3: cognitive skills to exercise critical thinking and judgement in developing new understanding S4: technical skills to design and use in a research project S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences **Application of Knowledge and Skills** A1: with initiative and judgement in professional practice and/or scholarship A2: to adapt knowledge and skills in diverse contexts A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

Complete lecture notes and exercise sheets will be provided.

6. Assessment

Exam/assignment/classwork breakdown					
Exam	**60**	**Assignment**	40	**Class work**	Enter %
Assignment due dates		**August 5th, 2022**	**August 26th, 2022**	**September 16th, 2022**	**October 21st, 2022**
Approximate exam date				**November 9th, 2022**	

Institution honours program details

Weight of subject in total honours assessment at host department	12.5%
Thesis/subject split at host department	**37.5%/62.5%**
Honours grade ranges at host department	
H1	**85% - 100%**
H2a	**75% - 84%**
H2b	**65% - 74%**
H3	**50% - 64%**

Institution masters program details

Weight of subject in total masters assessment at host department	Click here to enter text.
Thesis/subject split at host department	Click here to enter text.
Masters grade ranges at host department	
H1	Enter range %
H2a	Enter range %
H2b	Enter range %
H3	Enter range %