MTH5220 Martingales
Sem 1, 2023

Quiz

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1 A number X is selected uniformly at random in the interval $[-1,1]$. Let the events $A = \{X < 0\}$, $B = \{|X - 0.5| < 1\}$ and $C = \{X > 0.75\}$.

(a) Find the probabilities of B, $A \cap B$ and $A \cap C$.

$\mathbf{P}(B) = 3/4$, $\mathbf{P}(A \cap B) = 1/4$ and $\mathbf{P}(A \cap C) = 0$. □

(b) Find the probabilities of $A \cup B$, $A \cup C$ and $A \cup B \cup C$.

$\mathbf{P}(A \cup B) = 1$, $\mathbf{P}(A \cup C) = 5/8$ and $\mathbf{P}(A \cup B \cup C) = 1$. □

2 Suppose that X_1, X_2 are independent geometric with parameter p, where $p \in (0, 1/2]$. Find the p which maximises the probability of the event $X_1 = X_2$.

$$\mathbf{P}(X_1 = X_2) = p^2 \sum_{k=1}^{\infty} (1-p)^{2(k-1)} = \frac{p^2}{1-(1-p)^2} = \frac{p}{2-p}.$$

The latter is an increasing function on p, hence $p = 1/2$ is a maximum.

□

3 Suppose that $(X_i)_i$ are i.i.d. random variables, distributed as a Poisson(1). Find

$$\mathbf{E}\left[\prod_{i=1}^{n} X_i\right].$$

Using Independence, and the fact that each of the random variables have mean 1

$$\mathbf{E}\left[\prod_{i=1}^{n} X_i\right] = \prod_{i=1}^{n} \mathbf{E}[X_i] = 1.$$

□

4 Let $(S_n)_n$ be a simple random walk, i.e. $S_n = \sum_{i=1}^{n} X_i$, with $(X_i)_i$ being i.i.d. mean zero random variables each taking values in $\{-1, 1\}$. Find the (approximate) probability

$$\mathbf{P}(S_{10000} \geq 100).$$

Use Central Limit Theorem. This is approximated by the probability that a standard normal is larger or equal than 1. Using the normal table, the answer is 0.1587. $\square$