ACE Network Subject Information Guide

Random Matrix Theory, MAST90103
Semester 1, 2024

Administration and contact details

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Host institution	University of Melbourne
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Subject details

Handbook entry URL	https://handbook.unimelb.edu.au/subjects/mast90103
Subject homepage URL	https://canvas.lms.unimelb.edu.au/courses/197611
Honours student hand-out URL	Does not exist
Start date:	26th of February 2024
End date:	26th of May 2024
Contact hours per week:	3
Census date:	3rd of April 2024
Lecture day(s) and time(s):	Monday 1:00pm-2:00pm Wednesday 4:15pm-5:15pm Thursday 11:00am-12:00pm
Description of electronic access arrangements for students (for example, WebCT)	Zoom, Canvas/LMS webpage, Gradescope and Ed-Chatroom Platform

Subject content

1. Subject content description

Random matrix theory is a diverse mathematical tool. It draws together ideas from linear algebra, multivariate calculus, analysis, probability theory, group and representation theory, differential geometry, combinatorics and mathematical physics. It also enjoys a wide number of applications, ranging from wireless communication in engineering, to time series analysis in statistics, quantum chaos and quantum field theory in physics, to the Riemann zeta function zeros and prime numbers in number theory. A self-contained development of random matrix theory will be undertaken in this subject from various viewpoints.

2. Week-by-week topic overview

1st Week: Recalling Basics from Linear Algebra and Analysis and Gaussian Integrals
2nd Week: Likelihood Estimation, Loop Equations and Macroscopic Level Density
3rd Week: Level Repulsion and Wigner's Surmise as well as Involutions
4th Week: Classification of Symmetric Matrix Spaces and Wigner Ensembles
5th Week: Diagonalisation of Group-invariant Ensembles
6th Week: k-point Correlation Functions and the Method of Orthogonal Polynomials
7th Week: Determinantal Point Processes
8th Week: Local Spectral Statistics and Saddle-Point Approximation
9th Week: Gap-Probabilities
10th Week: Pfaffian Point Processes and the Cases of β=1,4
11th Week: Log Gases and Tricomi's Formula
12th Week: Matrix Sums and Products and the Concepts of Free Probability Theory

3. Assumed prerequisite knowledge and capabilities

Essential: Real Analysis, Vector Calculus, Complex Analysis, Linear Algebra

Desirable: Measure and/or Probability Theory, Group Theory and/or Algebra, Differential Geometry, Mathematical Physics and/or Statistics

4. Learning outcomes and objectives

-Gaussian random matrix models and their application in likelihood analysis and modelling covariance matrices in time series analysis;
-eigenvalue densities and the concept of eigenvalue repulsion;
-classification of random matrices ensembles;
-derivation of Jacobians for matrix transformations such as diagonalisations;
-joint eigenvalue densities and correlation functions;
-orthogonal polynomials and the concept of determinantal point processes;
-the log-gas picture;
-free probability theory and its application to matrix sums and products.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Identify the objectives of random matrix theory from the viewpoint of mathematical physics, and other areas of mathematics such as probability theory and mathematical statistics	K1,K2,S1,S2
Compute matrix Jacobians, apply the concepts of joint eigenvalue probability density functions, correlation functions, and spacing distributions, and understand their relevance to random matrix theory	K2,S1,S2
Demonstrate comprehension of how the symmetry classification is related to matrix (Lie-)groups	K1,K2,S2

Explain the basic ideas of the techniques of orthogonal polynomials, loop equations, moment method and free convolutions in the analysis of random matrices	K1,K2,S2
problem-solving skills: the ability to engage with unfamiliar problems and identify relevant solution strategies	S1,S2,S3,S4,A1,A2
analytical skills: the ability to construct and express logical arguments and to work in abstract or general terms to increase the clarity and efficiency of analysis	S1,S2,S3,S4,S5
time-management skills: the ability to meet regular deadlines while balancing competing commitments	A3,A4

Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills
S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

6. Assessment

Exam/assignment/classwork breakdown					
Exam	60%	**Assignment**	40%	**Class work**	0 %
Assignment due dates		22 March 2024	12 April 2024	26 April 2024	17 May 2024
Approximate exam date				First to third week in June	

Institution honours program details

Weight of subject in total honours assessment at host department	N/A

Thesis/subject split at host department	N/A
Honours grade ranges at host department	
H1	80-100 %
H2a	75-79 %
H2b	70-74 %
H3	65-69 %

Institution masters program details

Weight of subject in total masters assessment at host department	12.5 credit points of 200 credit points
Thesis/subject split at host department	20 credit points of 200 credit points
Masters grade ranges at host department	
H1	80-100 %
H2a	75-79 %
H2b	70-74 %
H3	65-69 %